JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 33, No. 2, March–April 2010

Engineering Notes

Synthetic-Waypoint Guidance Algorithm for Following a Desired Flight Trajectory

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DOI: 10.2514/1.46204

Nomenclature

N	=	proportional gain
R	=	closing range
R^*	=	virtual range
T_H	=	time horizon
V_a	=	true airspeed
V_w	=	synthetic-waypoint speed
x_c	=	output command
γ_C	=	commanded climb angle
γ_{ref}	=	reference climb angle
ΔT	=	sample time
λ	=	heading error
ψ_C	=	commanded heading angle
ψ_{ref}	=	reference heading angle

I. Introduction

THE development of aircraft guidance, navigation, and control systems has been a long-standing research area. Numerous methods relating to the enhancement of aircraft performance under various mission parameters have been developed in response to a need for more reliable and robust guidance systems. Current guidance systems applied to commercial, civilian, and unmanned aircraft rely on the knowledge of a flight path, specified by waypoints located in inertial space. Most missions are considered successful when the vehicle reaches the designated waypoint at which new commands are issued to the vehicle to proceed to the next waypoint. Two common types of conventional aircraft guidance are the direct-to-waypoint (DTW) and track-to-waypoint (TTW) methods in relation to path-following between designated waypoints. The DTW method simply issues heading commands to the vehicle based on the angular difference between the waypoint and vehicle. When the vehicle reaches the waypoint, the control system issues a new command to guide the aircraft to the next waypoint. The TTW method aims to follow the track between waypoints. In this guidance method the control system aims to minimize the lateral offset between the prescribed flight path and the aircraft's position, issuing heading commands that return the vehicle to the nominal flight path. The track method therefore places the additional constraint on a flight path that the vehicle must follow in order to reach the waypoint, rather than simply reaching the waypoint. However, both methods are far from optimal. This is evident in how the aircraft transitions between flight paths after reaching a waypoint. During these flight-path transitions, the aircraft will often overshoot the desired flight path to correct its track, particularly when the flight-path transition angle is acute. Various control strategies have been investigated to alleviate or minimize flight-path deviations. Such strategies include applying modern control methods such as receding-horizon control [1,2] and model predictive control [3] to anticipate flight-path changes and take control action before reaching a goal while maintaining adequate vehicle flight performance.

Missile guidance and control systems operate on similar principles to commercial, civilian, and unmanned aircraft guidance and control algorithms. The primary mission for missile systems is to intercept a moving target using information about the relative position and velocity between the pursuer and target. One of the first methods used in missile guidance was pursuit guidance (PG) [4–7]. The method operates by forcing the angular displacement error between a pursuer and its target to zero. Control commands scaled by a proportional factor of the current error are then issued to direct the pursuer along the line of sight (LOS) between the pursuer and target. PG solutions, however, do not consider the path taken or the levels of system performance required by the pursuer in reaching the target, resulting in a far-from-optimal solution. To address this problem of sub-optimality, additional parameters have been introduced to enhance missile performance. One method includes taking into account the motion of the commanded line of sight between the pursuer and target [4,7–10], issuing lateral acceleration commands based on tracking error and tracking error rate to the target. This approach has been shown to improve overall interceptor performance compared with conventional PG [4,7]. Another such method aims to modify the level of control the guidance algorithm possesses over the vehicle by adjusting the level of proportional gain. This is achieved by gain scheduling [11] to select gain values based on current interceptor states.

In addition to modifying internal missile guidance and control parameters such as variable gains and LOS rate estimation, mission performance can be enhanced by manipulating the trajectory taken by the pursuer to the targets. A good example of such a method is discussed in [12], in which a missile aims to exploit the aerodynamic benefits of high-altitude flight by tracking a virtual target at some initially high altitude that is not necessarily along the trajectory to the true target.

This Note discusses the development of a guidance law fusing the virtual-target concepts with those of pursuit guidance for implementation into an aircraft guidance system. This Note develops a path-following aircraft guidance algorithm that pursues synthetic waypoints using only a small set of guidance parameters, extending the virtual-target concept to complete aircraft guidance. The path is defined by the track between a minimal set of waypoints at specified locations, removing the need for a smooth path to be defined or the need for complicated path-switching logic or trajectory planning when a waypoint is reached. The synthetic waypoint travels along the path between waypoints, with the trailing aircraft traveling a smooth path generated through its own dynamics in following the synthetic waypoint. The guidance law is tested by varying guidance parameters, thus assessing vehicle sensitivity to and overall system performance of parameter variations. The following sections discuss the basic concepts of missile and aircraft guidance and provide a detailed description of the structure of the synthetic-waypoint guidance algorithm. A discussion on the implementation of the algorithm into the underlying aircraft control system will also be presented, followed by an analysis of the performance of the guidance algorithm in nonlinear simulation.

Received 30 June 2009; revision received 3 November 2009; accepted for publication 9 November 2009.

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II. Algorithm Overview

A. Pursuit Guidance

The theory of pursuit guidance is based on maintaining a direct heading toward a target by driving the angle between the forward axis of the pursuer and the LOS to the target to zero [7]. This is achieved by commanding the pursuing vehicle to maneuver at a rate equal to the LOS rate. As the target moves, the pursuer also moves to maintain a direct line to the target for interception. In its simplest form, the pursuit guidance algorithm can be expressed mathematically as

$$x_C = N\dot{\lambda} \tag{1}$$

where x_C is the output command, and N is a proportional gain constant that multiplies the LOS rate $\dot{\lambda}$. Figure 1 shows a typical pursuit engagement scenario between the pursuer and target. It can be seen that the pursuer aims to maintain a direct heading toward the target by aligning its direction of travel with the LOS to the target, by driving the LOS sight error angle λ to zero. As the target moves, so does the LOS, forcing the pursuer to adjust accordingly. As the closing range between the pursuing vehicle and the target reduces, the pursuer converges on the targets position.

The greatest benefit of applying pursuit guidance is in its simplicity. Only λ and $\dot{\lambda}$ need to be measured in order to generate sufficient control commands for an interception course. The gain term N is the only guidance parameter that needs to be defined, determining the input state's level of magnification. The gain term can be varied to allow for adequate flight performance, while enabling the vehicle to converge upon the commanded trajectory. However, the selection of N is heavily dependent on the aircraft current state, which results in varying levels of performance across the flight envelope if N remains unchanged. As a result, a schedule of gains needs to be specified to allow for consistent aircraft performance within the flight envelope.

B. Synthetic-Waypoint Guidance

1. Engagement Dynamics

In the SWG algorithm, a flight path is defined that designates the trajectory that the aircraft is to follow. The waypoints defining the flight path are specified in inertial space with reference to a fixed local frame. Following this, a synthetic waypoint is placed at the location of the first waypoint. The operation of the SWG algorithm is based on tracking the synthetic waypoint that travels along the designated flight path. The waypoint is considered synthetic, as its position on the flight path is a projection of the position at which the aircraft intends to be within a specified time horizon. The time horizon is a user-defined time interval by which the aircraft trails the synthetic waypoint. As the aircraft approaches the synthetic waypoint on the flight path, the synthetic waypoint repositions itself, forcing the vehicle to assume a new heading in pursuit.

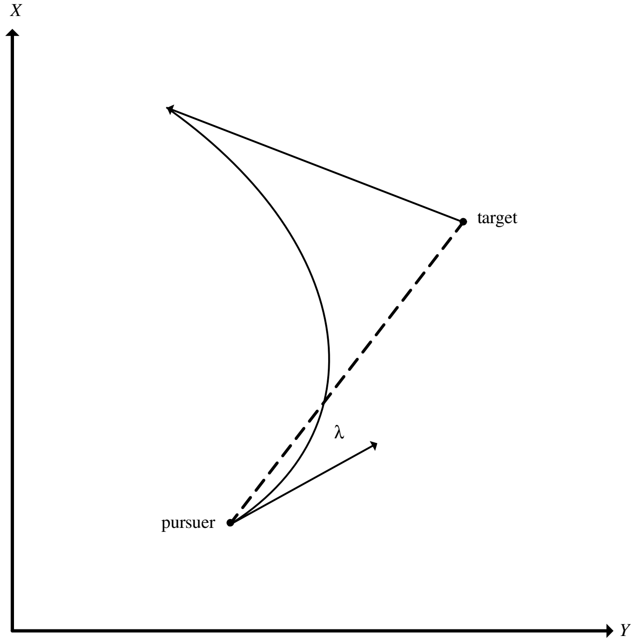

Fig. 1 Two-dimensional pursuer/target engagement.

Figure 2 illustrates the overall dynamics of a lateral engagement between the aircraft and the synthetic waypoint for a no-wind situation. Initially (Fig. 2a), the aircraft is a distance R from the flight path defined by the LOS between the vehicle and synthetic waypoint. The term ψ_C defines the commanded heading angle required by the aircraft to head toward the synthetic waypoint's current position. The synthetic waypoint is initially located at point 1. As the closing range R reduces during the engagement midcourse (Fig. 2b), the synthetic waypoint begins to move along the flight path between points 1 and 2 along a reference path with heading ψ_{ref} defined by the flight-path geometry. As the synthetic waypoint moves, its speed gradually increases, forcing the aircraft to change its heading with appropriate control to maintain a direct pursuit. As the closing range reduces further until a desired range is achieved, the speed of the synthetic waypoint along the flight path increases to match the speed of the aircraft. As a result, the aircraft continually pursues the waypoint along the flight path (Fig. 2c). The desired range that the vehicle aims to achieve behind the waypoint is a function of the aircraft current speed and time horizon. This will be discussed in further sections.

2. Guidance Laws

For the SWG algorithm to operate, certain constraints need to be applied to the dynamic behavior of the synthetic waypoint. Such constraints are applied to ensure correct tracking between the synthetic waypoint and the aircraft. The laws that define the dynamics of the waypoint are as follows.

The synthetic waypoint can only travel along the flight path between designated inertial waypoints. This constraint is necessary to ensure that the synthetic waypoint's movement is along the defined flight path. If this condition was not obeyed, then the guidance algorithm would be tracking a point that would lead the aircraft away from the flight path. Mathematically, this condition is expressed as

$$\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix}_i = \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix}_{i-1} + V_w \begin{bmatrix} \cos\gamma_{\text{ref}}\cos\psi_{\text{ref}} \\ \cos\gamma_{\text{ref}}\sin\psi_{\text{ref}} \\ -\sin\gamma_{\text{ref}} \end{bmatrix} \Delta T \tag{2}$$

where X_w, Y_w, and Z_w define the three-dimensional position of the moving synthetic waypoint in inertial space; V_w defines the speed of the synthetic waypoint; ψ_{ref} and γ_{ref} define the reference heading and climb angle of the flight path between fixed waypoints, respectively; and ΔT defines the integration sample used to update the position of the synthetic waypoint.

The minimum distance between the synthetic waypoint and the aircraft, referred to as the virtual range, must be set within the guidance algorithm. This constraint implies that aircraft can never be closer to the synthetic waypoint than the specified minimum range that will be defined as the virtual range R^*. Mathematically, this condition implies that

$$R \geq R^* \tag{3}$$

where R defines the closing range between the aircraft and current position of the synthetic waypoint.

The desired virtual range for a given flight condition is defined by the vehicle's current airspeed and time horizon. This constraint effectively defines the level of performance achievable by the aircraft. Mathematically, the virtual range can be expressed as

$$R^* = V_a T_H \tag{4}$$

where V_a represents the aircraft current airspeed and T_H specifies the desired time horizon for the vehicle to initiate a response to flight-path changes. The time horizon is analogous to prediction horizons

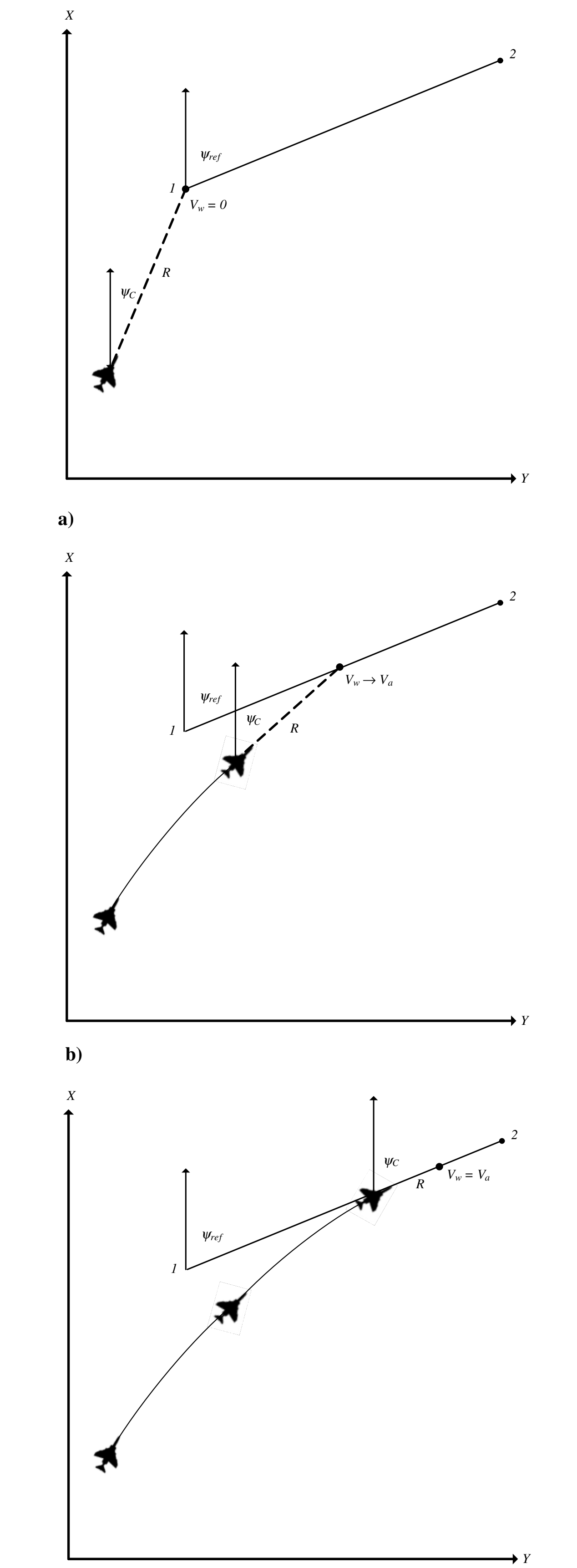

Fig. 2 Synthetic-waypoint engagement dynamics: a) initial engagement, b) midcourse, and c) flight-path acquisition.

discussed in most predictive control applications , which specifies a prediction horizon over which control action is taken to achieve a future goal. The time horizon in this application refers to the level of set-point prediction introduced into the system.

The speed of the waypoint is dictated by the ratio of the virtual range and current closing range multiplied by the aircraft current airspeed. This constraint is crucial to the overall guidance algorithm and ensures that the aircraft is always in pursuit of the synthetic waypoint. Mathematically, this condition is expressed as

$$V_w = V_a \frac{R^*}{R} \tag{5}$$

where V_w represents the speed of the synthetic waypoint along the reference flight path. This constraint deviates from the typical interceptor/target engagement scenarios in which the dynamics of the interceptor and target are independent. In this case, the dynamics of the synthetic waypoint and aircraft are intrinsically linked through V_a and R.

III. Aircraft Control

Aircraft flight control is driven by track and climb-angle commands calculated from the positional difference between the aircraft and synthetic waypoint through observation of LOS dynamics. The commands are then transformed into control inputs to aircraft control system. The formulation of the control system was performed using a linear quadratic regulator [13] design. It is important to note that the designed controller was kept constant throughout the analysis, as it is the effect of modifying SWG parameters on vehicle stability and performance that is being assessed, not the overall effectiveness of the controller. Ideally, a new controller would be designed to accommodate for SWG parameter changes.

Figures 3a and 3b show the relative position of the aircraft and the synthetic waypoint along the lateral and longitudinal axes. The coordinates (X_a, Y_a, and Z_a) and (X_w, Y_w, and Z_w) define the current positions of the aircraft and synthetic waypoint, respectively, in the inertial frame.

Consequently, the commanded heading and climb angles to the current synthetic-waypoint position can be evaluated as

$$\psi_c = \arctan\left(\frac{R_y}{R_x}\right) \tag{6}$$

$$\gamma_c = \arcsin\left(\frac{R_z}{R}\right) \tag{7}$$

where

$$R_x = X_w - X_a \tag{8}$$

$$R_y = Y_w - Y_a \tag{9}$$

$$R_z = -Z_w + Z_a \tag{10}$$

ψ_C is the commanded heading and γ_C is the commanded climb angle to the synthetic waypoint from the aircraft current position. Note that Eq. (6) operates a four-quadrant inverse.

Figure 4 shows the complete closed-loop system summarized in block-diagram form. The guidance block within the loop executes the SWG algorithm, receiving vehicle states from the aircraft dynamics to generate guidance commands that are also fed into the control and navigation system. The navigation system contains the defined flight-path information used to update the position of the synthetic waypoint, which is then fed back into the guidance block along with aircraft states. The control block receives guidance commands to generate control actions based on the current aircraft states relative to the synthetic waypoint. The controls generated are then used to guide the aircraft along the desired trajectory.

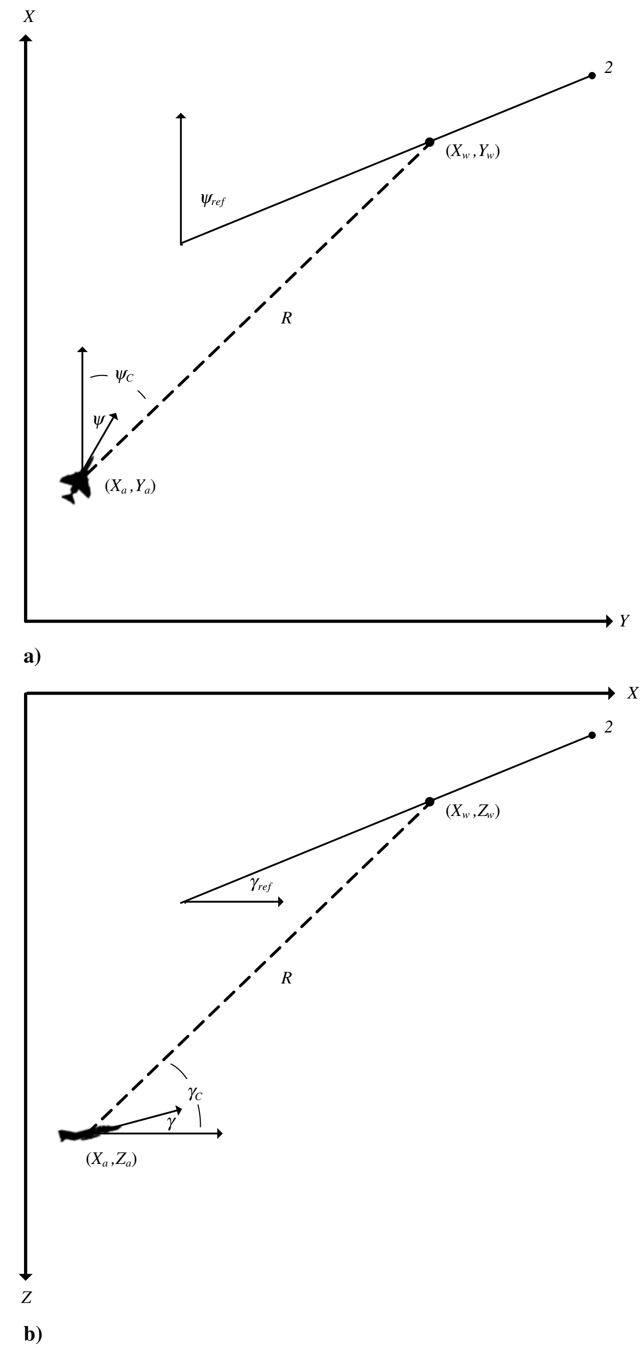

Fig. 3 Relative position of synthetic waypoint and aircraft in inertial frame: a) lateral and b) longitudinal.

IV. Simulation Results

This section demonstrates the SWG algorithm within a nonlinear aircraft simulation. The flight trajectory taken by the vehicle was defined in three dimensions and was known by the vehicle a priori.

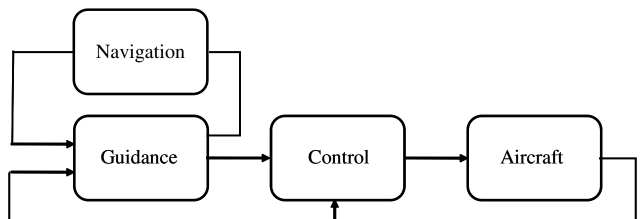

Fig. 4 Complete guidance loop structure block diagram.

The simulated aircraft is classed as a fighter trainer and was trimmed at an airspeed of 75 m/s (≈150 kt) at an altitude of 500 m.

Figure 5 shows the layout of the defined flight path in the inertial frame and the trajectory taken by the aircraft under different time horizons. The defined flight path consists of transition points requiring varying degrees of control activity to follow the desired trajectory. All the legs of the defined flight path were maintained at a constant altitude of 500 m. From the aircraft response, the trajectory taken by the vehicle follows the defined flight path closely in all cases, with minimal flight-path divergence using SWG.

Figures 6 and 7 shows an enhanced view around an acute-angle transition point. For the purposes of the simulation, the bank angle of the vehicle was limited to ±45°. Even though the flight path itself undergoes rapid changes in heading, the trajectory taken by the vehicle is smooth and rapidly reacquires the new flight leg once the maneuver has been completed. This is due to the fact that the synthetic waypoint transitions along a new flight path before reaching the true waypoint located at the apex. Since the aircraft is responding to changes in the synthetic-waypoint position, it begins to transition to the new flight leg before the apex is reached, avoiding rapid transition between legs, requiring high levels of control activity that may exceed the performance capabilities of the aircraft, particularly during acute flight-path changes (Fig. 7). In this region, it can be observed that the level of aircraft response is influenced by the time horizon. For the short time horizon (7.5 s), the new flight path is

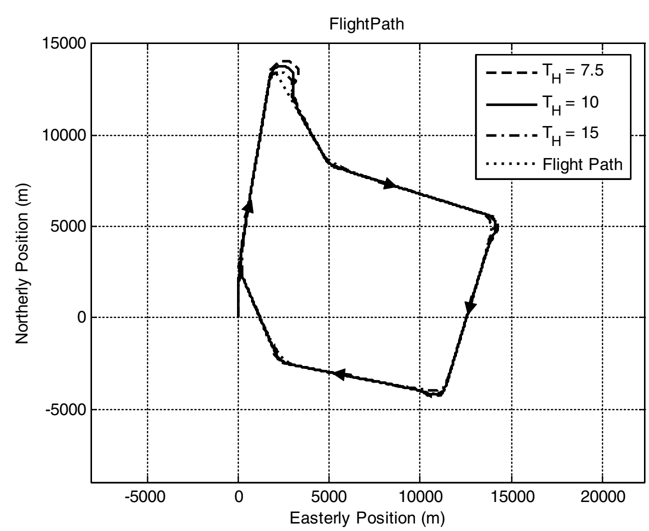

Fig. 5 Defined flight path and aircraft trajectory.

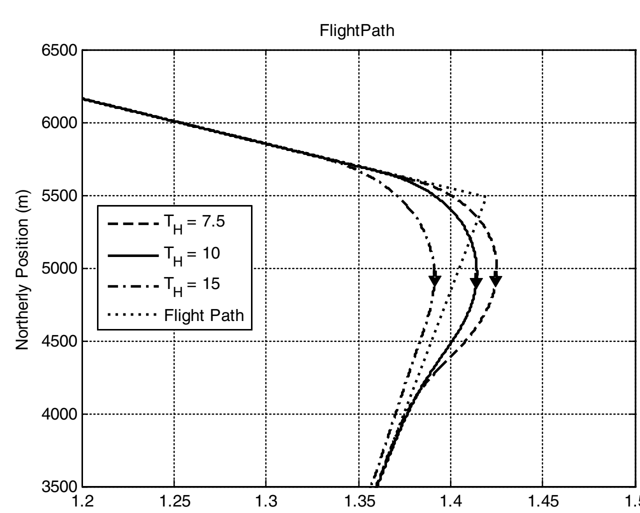

Fig. 6 Vehicle trajectory at transition point.

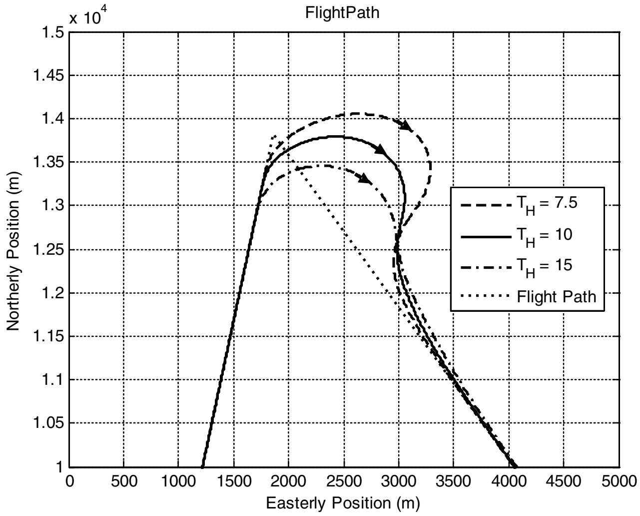

Fig. 7 Vehicle trajectory at acute transition point.

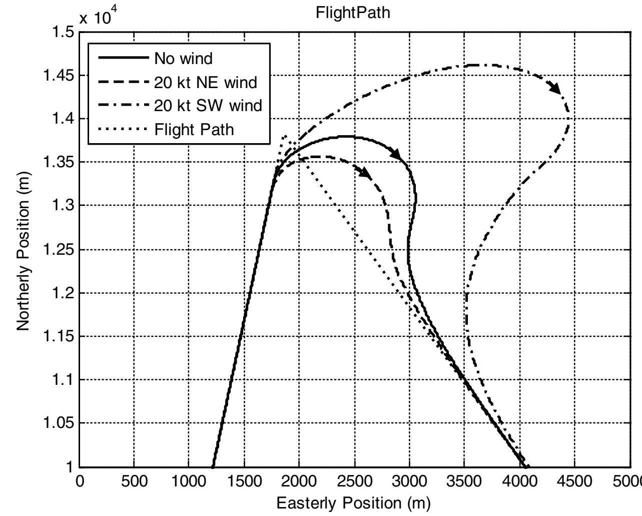

Fig. 9 Vehicle trajectory at acute transition point subject to 20 kt NE and 20 kt SW wind.

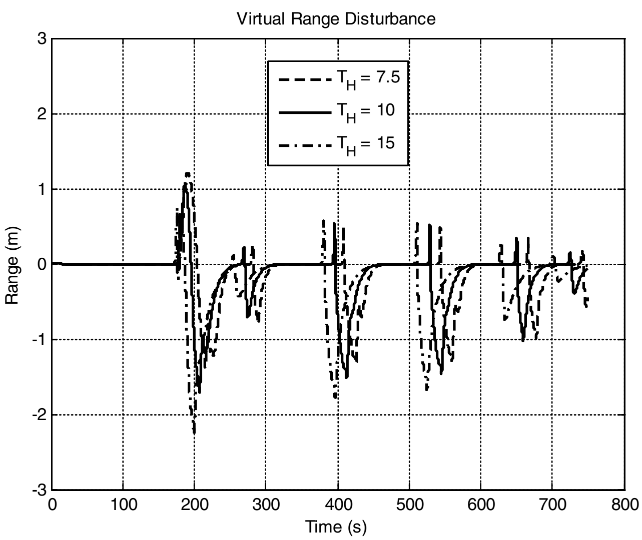

Fig. 8 Virtual range disturbance between aircraft and synthetic waypoint.

acquired faster than in the longer-time-horizon cases (10 and 15 s). However, this is at the expense of closed-loop stability, with the aircraft requiring a greater level of activity to return to the flight path.

Figure 8 compares the virtual range disturbance between the aircraft and the synthetic waypoint over the duration of the flight for the different time-horizon cases. It can be observed that the virtual ranges converge to the desired value formulated in Eq. (4) (562.5 m for the 7.5 s case, 750 m for the 10 s case, and 1125 m for the 15 s case given a constant airspeed of 75 m/s). The overall disturbance in virtual range remains relatively small for time-horizon changes, suggesting that the aircraft is maintaining a constant distance to the synthetic waypoint throughout the flight.

Figure 9 shows the trajectory taken by the aircraft in following an acute-angle turn while being subjected to a steady 20 kt northeasterly and 20 kt southwesterly wind. The time horizon was set at $T_H = 10$ s at an airspeed of 75 m/s. By observing the responses in both the wind and no-wind cases, it can be seen that the wind affects the aircraft trajectory during a maneuver, but the algorithm is stable enough to handle such disturbances and still follows the desired flight path closely throughout the duration of the flight. When the wind is acting from the NE, the aircraft is pushed to the SW, restricting the aircraft's divergence from the flight path when compared with the no-wind case. When the wind is acting from the SW, the aircraft is pushed to the NE, pushing the aircraft away from the desired flight path. However, the aircraft is still able to reacquire the current leg of the designated flight path once the maneuver is completed, demonstrating the SWG methods robustness to steady atmospheric disturbances.

V. Conclusions

This Note has demonstrated the development and implementation of a synthetic-waypoint guidance algorithm into a 6-DOF aircraft model. It has been shown through nonlinear simulations that the performance of an aircraft in following a specified trajectory is heavily dependent on guidance parameter changes: namely, time horizon. The results presented in the nonlinear simulations provided insight into the performance of the aircraft under various time-horizon conditions as well as demonstrating the algorithms ability to handle atmospheric disturbances. It demonstrated that an increase in time horizon provided an improved stability in flight-path convergence at the expense of response time.

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